MathletFactory:
A Component Framework and Authoring Environment for Mathematical Applets in Interactive eLearning Platforms

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Abstract:

New Media and New Technologies form the technical basis for a widespread development of visually-oriented teaching material as well as for the integration of interactive components into the learning process. Thereby, java applets – combining the potential of interactivity and visualisation – play a vital role in ambitious eLearning environments. For the mathematical education these two features are of a particular importance as they provide an outstanding opportunity to support the necessary changes in teaching methodology. Integration of applet technology also extends the opportunities of verification of user input important for intelligent training scenarios. The demand to allow concrete, individual experience in mathematics and its abstract objects and concepts through applets have far reaching consequences for the software design, in particular for the conceptual separation of an abstract object and its different representation forms, the interaction mechanisms between different mathematical objects, and the design of class libraries. The authoring process is another essential challenge. In this work we present the essential concepts of the MathletFactory *(subproject of the* Mumie *platform project).*

1 Introduction

Being one of the driving forces for the rapid technological progress, the role of mathematics in society is increasingly shifting towards its application in a wide variety of contexts. Highly advanced computer algebra and mathematical software packages open doors to new opportunities for mathematical modelling, simulation and visualisation of engineering and scientific processes. These programs take over routine actions but require a new quality of mathematical understanding in return since within these tools sophisticated algorithms come into place used in a “black-box”-like manner: the ability to judge the adequacy of a mathematical model is as urgently needed as the skill to overview and to roughly verify the results through simple approximations and background knowledge. The dramatic increase in mathematical power and the broadening of mathematical research methods necessitate a broad change in mathematical education – in particular to meet the demands of the growing groups of users from the different applied fields as e.g. engineering, computer sciences, and economics [4].

The demands posed upon mathematical education call for an extensive shift from instructivistic learning to more constructivistic scenarios [7, 9]. For this, interactivity and visualisation play a vital role in the academic education of mathematics: Visualisation of mathematical content and objects is an important tool in supporting fundamental comprehension of mathematical concepts. The use of visualisation as a tool for learning is not limited to the simple clarification of mathematical or scientific facts and data. Rather, it allows representation of the structural properties of objects and methods and trains spatial awareness and visual imagination. Visualisation is of particular interest when teaching newcomers and/or users from different applied fields since it provides a quick and descriptive first access to a previously unknown topic.

Interactivity of the knowledge pieces representing mathematical objects and concepts provides ready access to "experimental learning", which is considered to be one of the most effective forms of learning. Experimental learning ensures the active participation of the student in the learning process which is known to lead to a much quicker and longer-lasting success in teaching. In particular, to promote the independent and reflected *use* of mathematics this goal has to be transferred to the pedagogical models: following the main axiom of modern learning theories, the actual learning and understanding process has to be seen as a self-directed activity overcoming the passive reading of books or hearing lectures finally failing to reach the active part [4].

Additionally, applets are capable of supporting the realisation of intelligent training scenarios which facilitate explorative, self-directed, experimental learning styles: at this point, current knowledge systems do not include systematic and consistent approaches for a continuous integration of "intelligent" verification and validation mechanisms. The majority of existing eLearning platforms is limited to simple multiple choice input or similar, restricting the possible problems that can be given. Approaches of highly intelligent, user-adaptive feedback-mechanisms (e.g., a linguistic analysis of answers typed in freestyle) are currently under development, but have not been very successful so far. Here, adjustable applets contribute to expand traditional methods of validation. As a side effect, this approach leads to a new, more constructive formulation of problems which is of pedagogical advantage when teaching mathematics to students from applied fields.

2 MathletFactory – Technical Concept

The development of applets is both time-consuming and technically challenging. On the one hand, the required technical skills and knowledge cannot be expected from all teachers while on the other hand, it is the teachers who possess the necessary pedagogical and field-specific competencies. The programmers and developers, in contrast, possess the technical skills but often lack in field-specific and pedagogical competencies [3].

As a result, authoring environments are needed to effectively support teachers in the creation of (mathematical) applets: the MathletFactory represents an editing tool simplifying the development of new applets and the modification of existing ones. It is based on a large selection of pre-designed base classes that can be composed into applets. Thus, it acts as both a library of mathematical classes on the one hand and as an interactive development framework stressing re-usability of the single components on the other hand [8].

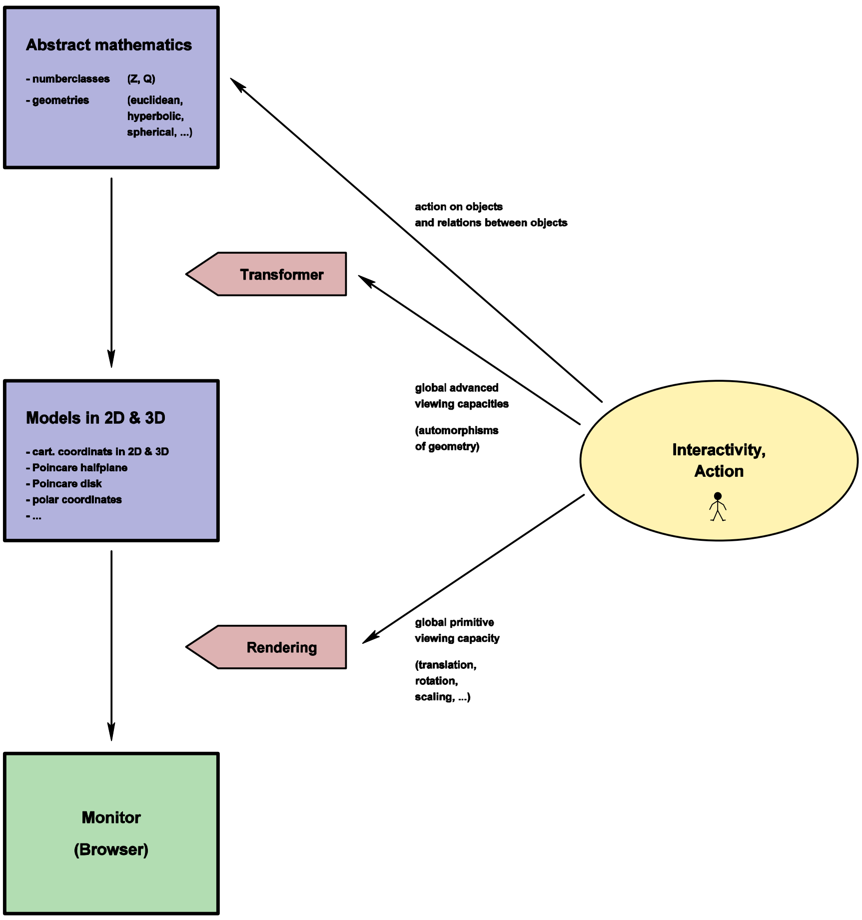

Figure 1: MathletFactory rendering chain.

The class structure of the MathletFactory is based on the specific logical structures and the relations within the field of mathematics. The actual classes account for the mathematical properties of the objects represented. A family of event-handlers provides mechanisms for visual interaction to facilitate the communication between the user and the objects themselves.

The internal design of the MathletFactory utilises an updating mechanism that modifies and adapts all objects and relations associated with a given event providing integrity and consistency of all relations and dependencies.

The architecture of the MathletFactory is characterised by the strict separation of the mathematical content itself on the one hand and the interaction and representation of that same content on the other hand: e.g., the actual mathematical objects are handled independently of the geometry used in their visual representation. This allows the generic visualisation of mathematical objects in different geometries, such as Euclidean, hyperbolic or spheric ones.

The actual rendering of the mathematical objects in a given geometry itself is separated from the mathematical modelling of the object, forming a third distinct layer: a model of controllers, handlers, and updaters facilitates the interaction between the user and the mathematical objects. User-triggered events (such as changing an object or its relations) are received by a controller and passed on to the respective handler. The handler will then change the actual object, while updaters ascertain that all dependent objects and relations are adapted accordingly.

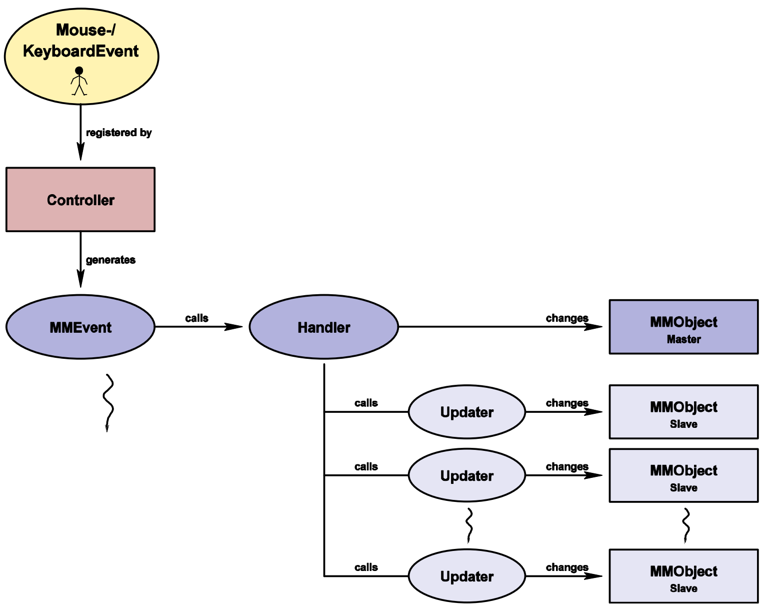

Figure 2: MathletFactory Updater Model.

3 MathletFactory from user's and author's perspective

All applets developed within the MathletFactory are characterised by a common basic structure, applied on the mathematical entities and their representations, the formal language processing capabilities of the symbolic representations and the generic display and interaction system.

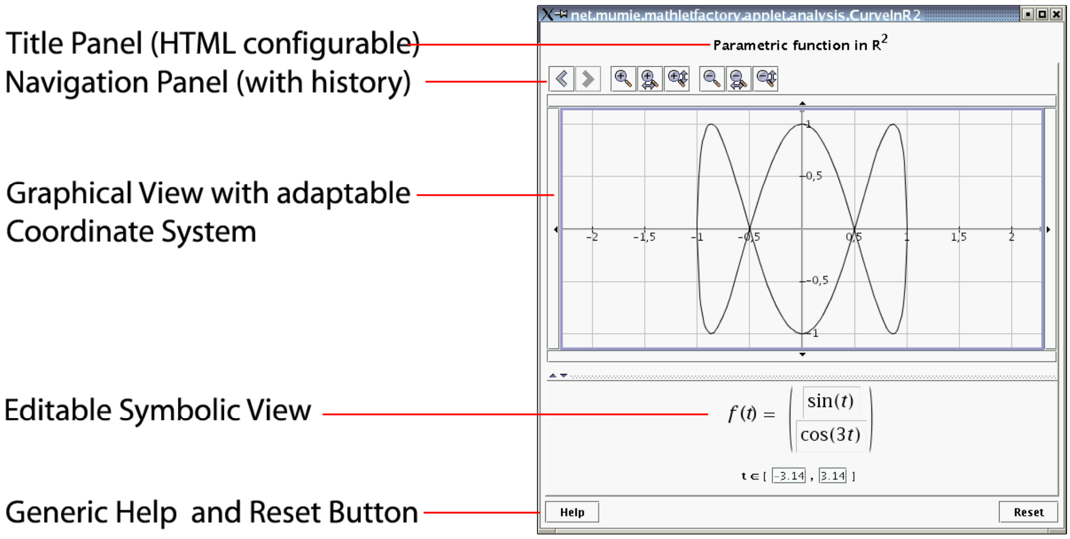

Figure 3: Standard design of applets, developed within the MathletFactory.

Figure 4 shows two demo-applets developed within the MathletFactory: the user may alter the initial condition by dragging the selected point and is immediately faced with the resulting mathematical solution.

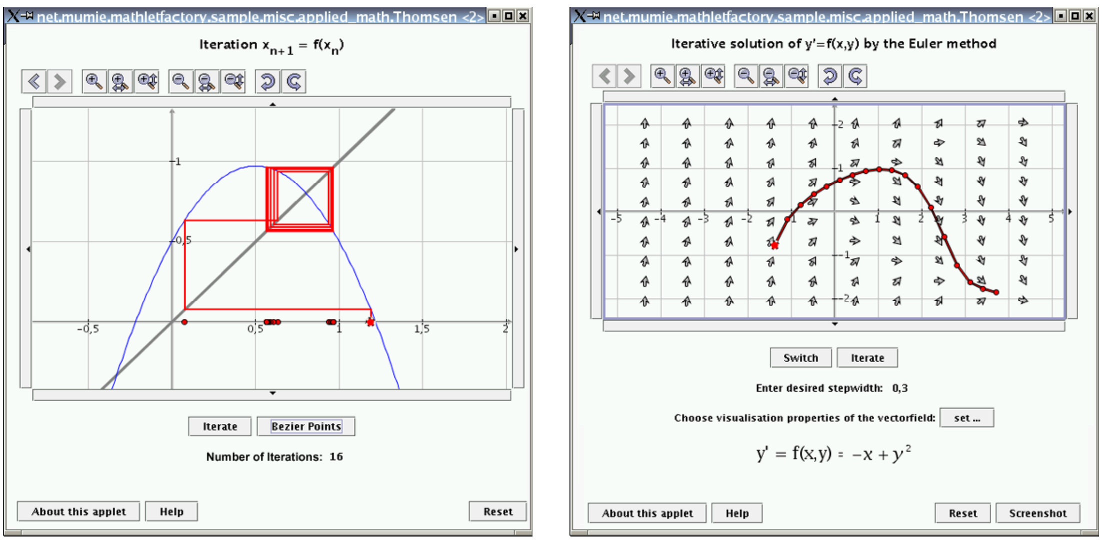

Figure 4: Demo Applets: iterated map (left), Euler integration (right).

The mathematical objects are displayed in different types of representation depending on the learning target and the given context: symbolic representations are of a particular importance to focus on the scientific mathematical language, iconic representations support the perception and the storage of the visual meaning of mathematical objects, and enactive representations allow "direct" interaction (for details see Bruners EIS schema of cognition psychology, [1]).

The authoring process is currently based on a MathletFactory-specific “Java-macro-language”. This concept, while representing a significant simplification over the implementation of all the necessary, actual code, still requires a certain, nontrivial degree of technical, basically Java-related, skills. Additionally, it does not support real-time mathematical experimentation, one of the more desirable applications for classroom deployment. As a result, we are currently developing visually-oriented authoring processes and composition mechanisms for the MathletFactory based on the software environment Oorange (www.oorange.de) which supports graphical programming, particularly for the framework of mathematical applications.

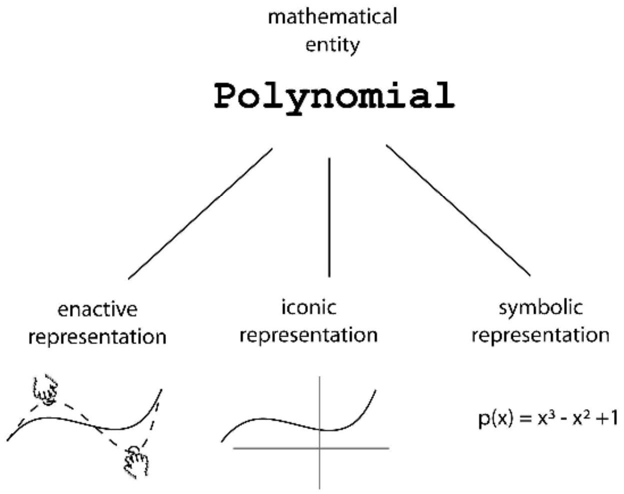

Figure 5: Different representations.

4 Summary and future prospects

The MathletFactory is a subproject of Mumie (Multimedial Mathematics Instruction for Engineers), funded by the German Ministry of Education and Research (BMBF). Mumie [6, 2] – currently in a prototype state – represents a multi-medial teaching and learning platform of the “Next Generation” optimised for mathematics and natural sciences and was developed by a cooperation of the TU Berlin, TU Munich, RWTH Aachen and University of Potsdam.

The MathletFactory offers not only a collection of mathematical classes but also a framework for the rapid development of highly interactive mathematical applets for both learning- and demonstration purposes. By separating the mathematical logic and its presentation and by providing a generic presentation and interaction architecture, a flexible and extensible system is now available, that allows the creation and customisation of applets to display complex mathematical relations. The applets built with the MathletFactory were already applied and evaluated in various academic and non-academic learning scenarios. A collection of more than 200 applets documents the breadth of mathematical and scientific subjects that are covered.

In terms of application of the MathletFactory one of the most important aspects is surely that of integration. While the applets were able to communicate with their enclosing web pages from the beginning, the latest development heads towards a deeper embedding of applets into comprehensive learning systems. Especially contributing to this goal is the implementation of intelligent and personalised training task applets, which allow the learner to interactively work on challenging assignments and receive direct feedback for his suggestions. A class evaluation of this is presently performed by the TU Berlin in application of the Mumie system.

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