Linear Algebra as a Bridge Course for First-year Engineering Students

Katherine Roegner

Department of Mathematics, Technische Universität Berlin, Berlin Germany

Abstract: Linear Algebra is frequently the first course in abstract mathematics and, as such, a hurdle for students in engineering fields. Beginning students are generally ill-prepared for the independent learning required by higher education, in which feedback to one concept comes well after the introduction of the next topics building upon it. Incoming students are often overwhelmed by the new learning environment typical of (especially large) colleges and universities. The anonymity of such institutions and the demands on students, especialy in the first year, can tempt students to copy homework solutions, a phenomenon that seems to have increased over the past years due to the internet.

The TUMULT (TUtorien MULTimedial) learning environment and its didactical implementation take advantage of technologiacal possibilities in order to deal with the difficulties described above. Feedback to procedural skills through online training exercises and support materials for lectures and tutorials allow instructors to focus more time on helping students to understand concepts and to improve their problem-solving skills. Electronically corrected homework sets in which each student receives the same type of problem, but with their own personal numbers, encourage students to discuss solution methods with each other, yet require students to solve problems individually. The time saved in grading many routine exercises can be invested in correcting a few in-depth problems. Through the introduction of prelearning, students are prompted as to how to prepare themselves independently for this course, thereby providing them with basic tools that may prove instrumental in the successful completion of their studies.

Background

Linear Algebra for Engineers at the Technische Universität Berlin (TU Berlin) is a required course for approximately 2000 incoming freshmen engineering students per semester. Typically, students weekly attend one of the four to five parallel lectures as well as a tutorial, the size of which varies between 35 and 55 students. In a traditional tutorial, the tutor solves sample problems at the board, whilst the students frantically copy everything down. Homework exercises with problems not substantially differing from the tutorial exercises are then solved and submitted in groups of two students. This group work, which is not to be confused with group learning in cooperative groups, such as described in [1], was introduced over a decade ago to cope with the growing number of students to be taught by a reduced number of tutors (due to budget cuts). In our experience, only one of the students in the group actually solves and writes up the solutions. This is confirmed by the typical 45 - 55 % passing rate for students who have acquired enough homework points over the semester to be admitted to the final examination. Due to the sheer numbers, the variety of mathematical abilities of the students (and the tutors), as well as the abtract nature of the course, Linear Algebra is considered a major hurdle for and by our engineering students. The outcome is disastrous, with only about 30 % of students enrolled in a tutorial at the beginning of the semester actually passing the final examination. Students who fail two written examinations are then required to take an oral examination. Many of these students turn out to have quite a bit of ability. So where are we failing our students?

In an effort to remedy this situation, an e-learning environment was developed for the course in linear algebra for engineers using the MUMIE (Multimedial Mathematics in Engineering Education) platform [2]. The key goals were to provide training modules with instantaneous feedback as well as to motivate students to actively participate in mathematics during the tutorials. Electronically corrected exercises were also integrated into the system to alleviate tutors from grading routine exercises so that their time could be invested more wisely, for example in correcting a theoretical problem for each student per week.

During the first three semesters in which this system was used, experimental tutorials were held in order to observe how students learn with the materials developed as well as to identify and find solutions for factors contributing to the failure of our students. The TUMULT (TUtorien MULTimediale) tutorials arose as a result of this study and have been an integral part of the course over the past three semesters. The puropose of this contribution is to report on the intentions and the preliminary outcomes of TUMULT.

Challenges and Strategies in the Mathematical Education of Engineers

During a three-semester study, several problems were identified in the way linear algebra was being taught to and learned by our students. This section is devoted to a discussion of the main points, most of which do not appear to be specific problems of the TU Berlin, as well as our approach to rectifying these shortcomings.

Preparing for and attending class

Students do not come prepared for the lecture. Reading the new material prior to the lecture in order to identify areas of confusion would substantially help students to better understand the lecture. Generally the students attend the lecture (if they even go at all) with the thought that they can understand mathematics by osmosis: attending the lecture equals understanding. If they do not understand the lecture, then it is because the lecturers are ill prepared, talk in mathematical jargon, etc., which may be true some of the time, but certainly not all of the time. Especially freshmen students are unaware that they need to take responsibility for their own learning at the university. Students who were good pupils in mathematics at the high-school level often have a distorted perception of their own mathematical abilities when they enter the university, as they were previously comparing themselves only with others at the same high school. In a poll of incoming freshmen, only 15 % said that their knowledge of mathematics was less than average [3] even though assessment skills testing showed otherwise. Furthermore, the expectations of university studies are much different at this level than what beginning students are used to.

Working through examples before the tutorial is for most students just as unthinkable as reading the material before the lecture. The outcome is an inefficient tutorial, with most of the time being spent on a review of the lecture. For this reason, many students opt to skip the lecture, resulting in them being even less prepared for the tutorial. The first exercises in the tutorial, for which tutors spend most of the time, are generally of a computational nature that lead to the theory. Due to time demands, the theoretical problems at the end of the tutorial are rushed through. The result is that students have great difficulties in solving theoretical problems in the final examination, especially when it comes to understanding the connections between concepts. Furthermore, as students are generally forced into the role of passive consumers during the tutorial due to time constraints, questions in the examination with a slightly different wording lead to confusion amongst the students.

To combat these problems, prelearning was introduced as an integral part of the course. Although this is not at all new a new concept, it is for our students at the TU Berlin. The main point here is that with an e-learning system, the prelearning problems can be controlled electronically, so that instructors are not required to do more grading. Students are required to solve two relatively simple problems that relate the new material to school mathematics at the beginning of the semester or to previous concepts in the course towards the end of the semester. For example, before introducing the Gaussian elimination process, students are required to solve systems of two equations in two variables which demonstrate the fact that a system of equations can have no, one or infinitely many solutions. They are asked to reflect upon which operations were permissible, preparing them for the ensuing discussion on Gaussian elimination in the lecture and tutorial. In a later stage, when determinants are introduced, students are prompted to make connections with invertibility and linear independence.

Lecturers who have previously held this course before prelearning times are amazed that students now answer questions posed during the lecture. The acceptance among students is higher when the lecturer ties the prelearning problems in with the lecture, for otherwise students see prelearning as busy work. One half of the students say that they feel like the prelearning problems help prepare them for the lecture. About one half find the prelearning problems too easy. It is not foreseen to beef up these problems in the near future, as it is not the intention to demotivate weaker students from the onset. Perhaps optional problems that are more challenging could help to motivate more advanced students.

Working ample exercises and getting timely feedback

As mentioned previously, only about one half of our students worked enough exercises to pass the examination. Students tended to exchange answers through the internet Forum provided, in which the answer was the focus, not the solution strategy. Furthermore, the feedback from the tutor came often two to three weeks after the introduction of a topic in the lecture, so that students were quite often unsure if they understood the material so that they sought help too late in the course.

TUMULT provides students with many possibilities for visualization as well as training concepts and computations online with feedback at the click of a button. Not only routine exercises, such as matrix multiplication, can be practiced. It is also possible for students to learn and test their knowledge on theory and connections, such as what a coordinate vector is or the geometric interpretation of an eigenvalue for a linear mapping from $\mathbf{R}^2$ to $\mathbf{R}^2$.

The training modules have also been integrated into the tutorials. Each of the three to four themes per week begins with a learning check to identify where students are in terms of their understanding. After the ensuing discussion, students then train in small groups at a computer (often their own laptops), freeing the tutor from the board to control student progress. Of course, tutors used to be able to pose a question and have students work on problems. The advantage with the electronic training is that students can ask for another question if they finish ahead of the others. We have also found that the new media sparks more interaction amongst students. What one cannot figure out alone can often be solved through a discussion with fellow students. Ideally the tutor should be more of a coach than the central omnipotent figure in the tutorial. Time constraints, however, do not always allow for this.

Each training module is equipped with a demonstration and training environment. The related electronic homework problem is often stated somewhat differently than the problem in the demo and training portions in order to guarantee a certain amount of transfer of learning. The problems in the training and homework portions are individualized and have a mask in which free form answers can be entered. Each student has the same type of problem, but each has their own randomly generated values. Students therefore communicate with one another about solution strategies, not just solutions. Exercises can be practiced and checked until enough confidence is gained for working the homework problem to be electronically graded. Copying, which used to be conceived as a major problem, is rendered useless.

The training possibilities are widely accepted amongst students. Polls taken [3] show that 85 % of our students use the online trainer on a regular basis. The electronic corrector is one of the least popular aspects of the TUMULT program, with only about a 45 % approval rate. The problem is that with free answers, it is difficult to award partial credit. If the automatic corrector recognizes that a plus-minus error was made, some points can be assigned. Other minor errors are not so easily detectable. One solution would be to require students to enter their steps in between, which could become annoying for students who are certain of their final answer. Furthermore, the prompt for the answer to a between step would also provide too many hints as to how the student should proceed and would require all students to use the same procedure for solving a problem, which is philosophically incompatible with TUMULT, in which we encourage students to develop and explore a variety of solution strategies.

Communicating mathematics

During the tutorials, students are expected to learn to communicate mathematics orally. Yet writing mathematics is also a task that is difficult, not only for engineering students. In the past, correcting final examinations was often frustrating. Many students would jot down a few numbers, and it was up to the graders to figure out what they were trying to say with them. A common question in tutorials was, “Okay. I think I understand. But how do I know what to write down?” It became clear that discussion and even presenting solutions for students is insufficient for developing their writing skills.

In order to develop written skills in mathematics, TUMULT students are required to turn in one written assignment per week, which is often an examination problem from a previous semester. In order to give students some orientation, the ideas set forth in Polya’s book *How to Solve It* [4] have been condensed into the following scheme: First students should reformulate the problem in their own words, bringing in data and definitions where necessary. They should then relate their plan of action. Which theorems are helpful? Which algorithms will they use? After carrying out their plan, they should then check their solution. In order to get students started, various examples are given to illustrate the scheme.

The time tutors save in not having to grade four problems for each group of two students is now used for grading this one written problem per student each week. It is expected that corrections also contain comments, not just checkmarks and x's. Discussions between the tutor and individual students about written work are highly encouraged. Even a five minute consultation before or after a tutorial can be extremely beneficial for clearing up a misconception or for motivating students.

The first semesters in which the Polya scheme was introduced, students were very appreciative. It is helpful for students, as long as one is not a slave to the scheme and sees it as an *orientation*. Especially the reformulation of the problem annoys students, the major argument being that they will not have time in the examination to reformulate the problem. It is hard to convince them that by practicing reformulation consciously now could help them subconciously in the examination, and the time it would take to convince them could be wisely used elsewhere. This semester, on the other hand, the assistants have been very dilligent in having the tutors control the form as opposed to focusing on the mathematics. Needless to say, the Polya scheme is not very popular this semester and will need to be redirected for the next.

Tutor support

Previously, tutors were provided with an exercise set including solutions. The preparation for the tutorial generally consisted of perusing the solutions and jotting down a few remarks as to which "tricks" should be presented additionally.

The TUMULT tutorials are much more demanding on tutors. Training is provided for incorporating multimedia into the classroom as most have no experience in this direction. Apart from a two-day schooling for our tutors, a weekly meeting is held to give them insight on how to use the training modules and to discuss other aspects of the learning environment.

In order to aid them in their preparation for each tutorial, suggestions are given for which definitions and theorems are available in the learning environment and at which point in the tutorial it would be useful to project them. The training modules are also specified along with questions to help them get discussions going between students, along with tips concerning moderating discussions when appropriate. This is perhaps more challenging for tutors than incorporating the multimedia into their tutorial, for it is much easier to give answers than to ellicit them. It is this aspect that will challenge instructors in the upcoming semester.

Discussion

The TUMULT learning environment, including its intricate didactical concept, has been an integral part of the course Linear Algebra for Engineers at the TU Berlin for the past three semesters. As with any system, there are obstacles that need to be overcome for the TUMULT learning environment to succeed in the long run. First and foremost it is crucial to gain student *and* faculty acceptance for this type of learning. Students do not like the picky electronic grader, and faculty members worry that they themselves will have to begin using multimedia in their lectures. Resources are scarce, so that time slots in computer laboratories need to be negotiated each semester until ample classrooms are equipped with WLAN and multiple electrical outlets, provided there are enough students willing to bring their laptops to the tutorial.

On the other hand, our 2000 students each semester come better prepared to the lectures and tutorials, which enables them to engage meaningfully in mathematical discussions. They take a more active and independent role in the tutorials, so that the tutor becomes more of a learning coach as opposed to the presenter of solutions. Not only has the oral communication of our students improved, their written work in the examinations is far beyond that of students from just a few semesters ago. We expect that the success rates of our students will begin rising once the introductory phase has been closed.

References

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